Math 201 — Fall 2011–12
Calculus and Analytic Geometry III, all sections
Final Exam, January 21 — Duration: 2 hours 15 minutes

GRADES:

Problem	1 (/15)	2 (/15)	3 (/15)	4 (/11)	5 (/14)	6 (/15)	7 (/15)
Part a							
Part b							
Part c							
Total							

GRAND TOTAL: GRADE:

YOUR NAME:

YOUR AUB ID#:

PLEASE CIRCLE YOUR SECTION:

Section 1 MWF 3, Kobeissi Recitation F 11	Section 2 MWF 3, Kobeissi Recitation F 5	Section 3 MWF 3, Kobeissi Recitation F 4	Section 4 MWF 3, Kobeissi Recitation F 10
Section 5 MWF 10, Abi-Khuzam Recitation T 11	Section 6 MWF 10, Abi-Khuzam Recitation T 3:30	Section 7 MWF 10, Abi-Khuzam Recitation T 5	Section 8 MWF 10, Abi-Khuzam Recitation T 2
Section 9 MWF 11, Brock Recitation T 12:30	Section 10 MWF 11, Brock Recitation T 2	Section 11 MWF 11, Brock Recitation T 11	Section 12 MWF 11, Brock Recitation T 3:30
Section 13 MWF 2, Nahlus Recitation Th 11	Section 14 MWF 2, Nahlus Recitation Th 3:30	Section 15 MWF 2, Nahlus Recitation Th 8	Section 16 MWF 2, Nahlus Recitation Th 5
Section 17 MWF 8, Makdisi Recitation F 2	Section 18 MWF 8, Makdisi Recitation Th 8	Section 19 MWF 8, Makdisi Recitation Th 2	Section 20 MWF 8, Makdisi Recitation Th 3:30
Section 21 MWF 1, Raji Recitation M 8	Section 22 MWF 1, Raji Recitation M 9	Section 23 MWF 1, Raji Recitation M 4	
Section 24 MWF 10, Egeileh Recitation F 11	Section 25 MWF 10, Egeileh Recitation F 2	Section 26 MWF 10, Egeileh Recitation F 3	

1. (5 pts each part,15 pts total)

(a) Use the Sandwich theorem to find the following limit

$$\lim_{n\to\infty} \frac{\frac{1}{1}+\frac{1}{2}+\cdots+\frac{1}{n}}{3\ln\sqrt{n}}.$$

(b) (UNRELATED) Find, with justification, all values of p for which the following series is convergent

$$\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\sin\left(\frac{1}{\sqrt{n}}\right)\right)^{p}.$$

continued...

(c) (UNRELATED) Compute the n^{th} partial sum S_n of the following series, and use it to find, according to the values of c, the sum of the series

$$\sum_{k=0}^{\infty} \frac{c^{k+1} - c^k}{(c^k + 3)(c^{k+1} + 3)}, \qquad c > 0.$$

2. (5 pts each part, 15 pts total)

(a) Consider the function

$$f(x,y) = \frac{xy^2}{3\sin^2 x + y^2}, \quad \text{if } (x,y) \neq (0,0)$$

Prove that $f(0,0)$ may be defined in such a way that f becomes continuous at $(0,0)$.

(b) If in part (a) $f(0,0)$ has been defined correctly, prove that f is not differentiable at $(0,0)$. For this part, you may use without proof that $f_x(0,0) = f_y(0,0) = 0$.

continued...

(c) (UNRELATED) By about how much will $h(x, y) = \ln(x^2 + y^2 + z^2)$ change if the point $P(x, y, z)$ moves from $P_0(1, 1, -1)$ a distance of $ds = 0.1$ unit in the direction of the vector $3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$?

3. (5 pts each part, 15 pts total)

(a) Sketch the region of integration and evaluate the double integral

$$\int_1^{15} \int_0^{1/y} y e^{xy} \, dx dy.$$

(b) Evaluate the double integral

$$\int_0^{32} \int_{x^{1/5}}^{2} \frac{dy dx}{y^6 + 1}.$$

continued...

(c) Evaluate the integral

$$\int_0^\infty e^{-x^2}\,dx.$$

4. (11 pts total: 5 pts for (a), 3 pts for (b), 3 pts for (c))

(a) Find the volume of the solid bounded below by the surface $z = 1$, and above by the surface $x^2 + y^2 + z^2 = 4$.

(b) If the density is $\delta(x, y, z) = z$, set up but do not evaluate, a triple integral with $dV = dxdydz$, giving the mass of the solid in part (a).

continued...

(c) Set up but do not evaluate the integral in part (b) in spherical coordinates.

5. (14 pts total: 5 pts for (a), 5 pts for (b), 4 pts for (c))

(a) Let $f(x) = \frac{1}{x+3}$. Find the Taylor series expansion of f about $a = 1$,(i.e., centered at $a = 1$), and use it to find $f^{(n)}(1)$.

(b) (UNRELATED) Find the maximum and minimum values of the function $f(x, y) = x^2 + y^2 + x - y$ on the curve $x^2 + y^2 = 2$.

continued...

(c) (UNRELATED) Use the transformation $x = au, y = bv, z = cw$ to find the volume of the region $R = \{(x, y, z) : \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1\}$. Here a, b, c are positive constants.

6. **(5 pts each part, total 15 pts)** Consider the region $D = \{(x, y) : x^2 + y^2 \leq 4, y \geq -1\}$, and the vector field $\mathbf{F}(x, y) = -y\mathbf{i} + x\mathbf{j}$ in D.

(a) Using appropriate parametrizations of the boundary, evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the positively oriented boundary of D.

(b) Use Green's Theorem to evaluate the integral in part (a).

continued...

(c) Let $\mathbf{G}(x, y) = \frac{-y}{x^2+y^2}\mathbf{i} + \frac{x}{x^2+y^2}\mathbf{j}$ be another vector field defined away from $(0, 0)$. Evaluate $\int_C \mathbf{G} \cdot d\mathbf{r}$ on the same curve C as before.

7. **(5 pts each part, 15 pts total)** Consider the vector field

$$\mathbf{F}(x, y, z) = (\frac{y}{1+x^2y^2})\mathbf{i} + (\frac{x}{1+x^2y^2} + e^z \cos y)\mathbf{j} + (e^z \sin y)\mathbf{k},$$

and let S be any curve in the first octant, starting at $A(\frac{1}{\pi}, \pi, 1)$ and ending at $B(\frac{2}{\pi}, \frac{\pi}{2}, \ln 3)$.

(a) Is F a conservative field in the first octant $\{(x, y, z) : x, y, z > 0\}$? Prove your answer.

(b) Evaluate the line integral $\int_S \mathbf{F} \cdot d\mathbf{r}$.

continued...

(c) (UNRELATED) Suppose $f(x, y)$ satisfies $f_{xx} + f_{yy} = 0$ in a domain D with positively oriented boundary C. Prove the following identity:

$$\int_C f\nabla f \cdot \mathbf{n} ds = \int\int_D |\nabla f|^2 dA.$$